Int J Theor Phys (2009) 48: 3257–3261
DOI 10.1007/s10773-009-0112-0

On Sciama's Machian Cosmology

Marcelo Samuel Berman

Received: 25 May 2009 / Accepted: 27 July 2009 / Published online: 1 August 2009

Abstract We handle Sciama's cosmological theory, extending a previous paper by Berman (Astrophys. Space Sci. 318:269–272, 2008), and calculating radiation processes, in Machian, Sciama's and general relativistic cosmological gravitation.

We find that the dipole or quadrupole formulae, when applied to Cosmology, yield the same constant radiated power for the Universe, as in a previous Machian calculation (Berman in arXiv:0806.1766v2).

We consider consequences for the growth of entropy in the Universe.

Keywords Einstein · Brans-Dicke · Newton · Gravitation · Mach · Sciama

1 Introduction

Amidst several alternative theories of Gravity, which modify General Relativity, there stands an electrodynamical-type gravitational theory, put forward by Sciama [14]. The idea behind inertia, would be, according to Sciama, that the Universe obeys Mach's principle, i.e., at each point of space, the total force acting on a particle, is null, being composed by the second Newtonian law of force $m\vec{a}$, summed with a negative equal inertia force, which would be originated from the rest of the Universe and applied at that point. Thus, the total energy of the Universe would also be zero. The interaction of the Universe in a local point, would be made through linearized equations originated from Maxwellian-type fields [1]. Berman [9], has detailed the cosmological consequences of Sciama's theory, and showed that the Universe has expansion plus rotation, while the angular speed is inversely proportional to the scale-factor (the "radial" coordinate), and the Universe obeys Brans-Dicke relation in the form of the equality,

$$G\frac{M}{c^2R} = \gamma \sim 1, \quad \text{where } \gamma = \text{constant}. \tag{1}$$

M.S. Berman (✉)
Instituto Albert Einstein/Latinamerica, Av. Candido Hartmann, 575, # 17 and 80730-440, Curitiba, PR, Brazil
e-mail: msberman@institutoalberteinstein.org

The purpose of the present letter is to show that the Machian Universe, as described by Berman [2–7], matches Sciama's linearized theory of "electrodynamical" gravitation; that (gravitational) radiation yields the same constant power, when the whole Universe is considered; and that the entropy grows while keeping the radius proportional to the inverse square of absolute temperature.

The research on the origin of Inertia is a problem that involved passionately a number of physicists especially from the crisis of Classical Physics and from the birth of General Relativity. Einstein himself underlined the importance of his meditation on this argument in the development of his theory of gravitation. Moreover he formulated precisely his reflections on inertia origin in the Mach's Principle—that in its simpler form says that inertia properties of matter are determined in some manner by the other bodies of the Universe. Even though with the development of General Relativity, Einstein rejected explicitly his first considerations on Inertia, the Principle represented, in a compact form, a research project that guided many scientists in the development of gravitational theories alternative to General Relativity (for example Brans-Dike Theory)—with the following cosmological implications—and the reformulations of Classical Mechanics based on Mach's reflections on Inertia (see for example the model proposed by Shrödinger).

2 On Berry's Machian Argument

Berry [11] has posed a Machian query. Consider a body of mass m, acted on by a large one M located at a distance r, while the large mass has an acceleration $\vec{a}$ relative to the small one. In order to satisfy Mach's principle, the force exerted on the small mass by the larger, must contain a part proportional to $m\vec{a}$. By means of dimensional analysis, we find that the correct force should be proportional to $m\vec{a}$ and also to other terms: M, r, G and c, at some powers. According to Newton's third law, the power of M and m must be the same, i.e., they occur symmetrically in the force equation. The solution is,

$$\vec{F} = -GM\frac{m}{c^2 r}\vec{a}. \tag{2}$$

This looks like the force whose acceleration measures mutually accelerated charges. For the gravitational case, Sciama has given a name to it: law of inertial induction. For one thing, we may understand from the analogy, that if electromagnetic radiation is possible, then we would also have gravitational radiation.

If law (2) is to be applied to the distant masses of the Universe, in the Machian picture, and if Newton's second law should be valid, we need the following Brans-Dicke relation to be valid:

$$\frac{GM}{c^2 r} = 1, \tag{3}$$

so that,

$$\vec{F} = -GM\frac{m}{c^2 r}\vec{a} = -m\vec{a}. \tag{4}$$

This section was a digression on a Berry's argument. It must be said that from formula (4), we have the same kind of zero-total force applied to each and all particles in the Universe, (inertial force plus gravitational force, equals zero) so that we retrieve a zero-total energy of the Universe.

3 Gravitational Radiation in Sciama's Machian Model

The Machian postulates are, *sphericity* (the Universe resembles a "ball" of approximate spherical shape), *egocentrism* (each observer sees the Universe from its center) and *democracy* (each point in space is equivalent to any other one—all observers are equivalent) [10].

Consider the rotating and expanding Universe. The angular speed is given by,

$$\omega = \frac{c}{R}. \tag{5}$$

The reason for the above formula, is that, if we suppose that the Universe has constant zero-total energy, and is rotating, we would write the energy equation as:

$$E = 0 = Mc^2 - G\frac{M^2}{2R} + \frac{L^2}{MR^2}. \tag{6}$$

In the above, the inertial energy is represented by the first term in the r.h.s. of (6) followed by the potencial energy and rotation terms, where L is the angular momentum of the Universe. We need such term in order to explain the Pioneer anomaly [4], which was an anomalous constant deceleration which Berman says to be caused by the rotation of the Machian Universe, appearing as a centripetal one, and implying that the angular speed is given by relation (5). The reason is that we find the Brans-Dicke "generalised" relations, which solve (6), namely,

$$G\frac{M}{c^2R} = \gamma \sim 1, \tag{1}$$

$$\frac{L^2}{c^2MR^2} = \gamma', \tag{7}$$

where $\gamma' =$ constant, is that there is no other solution with time-invariant zero-energy equation (see (6)), that allows time-varying $R = R(t)$. In fact, we find from (7) and (1), that $L \propto R^2$. On the other hand, we know from Newtonian physics that $L \cong RM(\omega R)$. From the two last relations, we find necessarily a relation of type (5).

If we further calculate the power produced in the rotational motion of the "ball", we find,

$$P = \tau\omega = F_t R\omega = (M\alpha R)R\omega = MR^2\alpha\omega, \tag{8}$$

where τ is the torque of the tangential force F_t, which produces angular acceleration α.

From (5) we find,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{c}{R}\right) = -cR^{-2}\dot{R} = -c^2R^{-2}, \tag{9}$$

where we have made use of the Machian relation for the radius of the causally connected Universe,

$$R = ct. \tag{10}$$

At last, we obtain,

$$P = -c^2M\omega = -\gamma\frac{c^5}{G}. \tag{11}$$

We would have obtained the same result, by introducing the equivalent of the electrodynamical Larmor power formula [13], which yields the power radiated by an electric dipole, or an accelerated charge. The equivalent gravitational law is,

$$P_{\text{Larmor}} = \frac{2}{3}\left[\frac{GM^2}{c^3}\right]a^2, \tag{12}$$

where a is the acceleration. For the rotating case, $a = \omega^2 R$, and then, relation (12) becomes,

$$P_{\text{Larmor}} = \frac{2}{3}\left[\frac{GM^2}{c^3}\right]\omega^4 R^2 = \frac{2\gamma^2}{3}\left[\frac{c^5}{G}\right]. \tag{13}$$

Both formulae ((11) and (13)) are pretty similar. It sounds as if Machian Universe is akin with Sciama's linearized gravitation. It is expected that the power-loss will be radiated as gravitational waves, as we shall discuss in the next section.

4 Quadrupole and Dipole Radiation

Einstein's quadrupole radiation formula [15], is of the type,

$$P_{\text{einst}} \approx \frac{G}{c^5}\omega^6 Q^2 \approx \frac{G}{c^5}\omega^6[M^2R^4]. \tag{14}$$

For our Machian Universe, the above will yield,

$$P_{\text{einst}} \approx \frac{c^5}{G}\gamma. \tag{15}$$

The formula for electrodynamical radiation depended on the dipole term's second time derivative, while the gravitational Einstein's radiation depends on the third time derivative of the quadrupole term. Nevertheless, for the Universe, we obtain the same result. Observe that we are in face of a constant radiating power, as was some time ago expressed by Berman [8]. The fact, that for the Machian Universe, the dipole and the quadrupole power formulae coincide, does not mean that we may just take one instead of the other in local situations.

5 Temperature of the Universe. Entropy

The power of a black-body radiator is given by Halliday et al. [12]:

$$P_{bb} = \sigma A T^4, \tag{16}$$

where A represents the radiating surface, at temperature T and σ is a constant.

For the Universe, we would find, on (16) with (11),

$$4\pi R^2 \sigma T^4 = \frac{c^5}{G}. \tag{17}$$

This results in the dependence of R with T^{-2}. This relation was found by Berman in several papers and books (see for instance, [2, 3, 7]). Now, let us calculate the entropy,

$$dS = \rho(4\pi R^2 dR)T^{-1}, \tag{18}$$

where ρ stands for the energy density of radiation, i.e.,

$$\rho = aT^4, \tag{19}$$

so that,

$$S = \frac{4\pi}{3} aT^3 R^3 \propto R^{\frac{3}{2}}. \tag{20}$$

We have found that the entropy of the Universe grows with $R^{\frac{3}{2}}$. This is a result of Sciama's theory, albeit Mach's theory. Berman has arrived to this formula in other cases [2, 3, 7, 10].

6 Conclusions

Sciama's linear theory, has been expanded, through the analysis of radiating processes, thus, extending a previous paper of the present author [9].

Larmor's power formula, in the gravitational version, leads to the correct constant power relation for the Machian Universe. However, we must remember that in local Physics, General Relativity deals with quadrupole radiation, while Larmor is a dipole formula; for the Machian Universe the resultant constant power is basically the same, either for our Machian analysis or for the Larmor and general relativistic formulae.

Acknowledgements I thank for the encouragement by Paula, Albert, and Geni and also to Marcelo F. Guimarães, Nelson Suga and Antonio F. da F. Teixeira.

References

1. Arbab, A.I.: Quantum universe and the solution to the cosmological problems. GRG **36**, 3565 (2004)
2. Berman, M.S.: Introduction to General Relativity and the Cosmological Constant Problem. Nova Science, New York (2007)
3. Berman, M.S.: Introduction to General Relativistic and Scalar-Tensor Cosmologies. Nova Science, New York (2007)
4. Berman, M.S.: The pioneer anomaly and a Machian universe. Astrophys. Space Sci. **312**, 275 (2007). Los Alamos Archives, arXiv:physics/0606117v2
5. Berman, M.S.: A general relativistic rotating evolutionary universe. Astrophys. Space Sci. **314**, 319–321 (2008). Los Alamos Archives, arXiv:0712.0821v2
6. Berman, M.S.: A general relativistic rotating evolutionary universe—Part II. Astrophys. Space Sci. **315**, 367–369 (2008). Los Alamos Archives, arXiv:0801.1954v2
7. Berman, M.S.: A Primer in Black Holes, Mach's Principle and Gravitational Energy. Nova Science, New York (2008)
8. Berman, M.S.: Gravitons, dark matter, and classical gravitation (2008). To be published by American Institute of Physics, Proceedings from a Symposium on Gravitational Waves, to be held in September 2008, in Greece. Los Alamos Archives, arXiv:0806.1766v2
9. Berman, M.S.: On the Machian origin of inertia. Astrophys. Space Sci. **318**, 269–272 (2008). Los Alamos Archives, arXiv:physics/0609026v2
10. Berman, M.S.: Entropy of the universe. Int. J. Theor. Phys. **48**, 1933 (2009). doi:10.1007/s10773-009-9966-4. Los Alamos Archives, arXiv:0904.3135v2. See also: Entropy of the universe and standard cosmology. Int. J. Theor. Phys. **48**, 2286 (2009). Los Alamos Archives, arXiv:0904.3135v2
11. Berry, M.V.: Principles of Cosmology and Gravitation. Adam Hilger, Bristol (1989)
12. Halliday, D., Resnik, R., Walker, J.: Fundamentals of Physics, 8th edn. Wiley, New York (2008)
13. Reitz, J.R., Milford, F.J., Christy, R.W.: Foundations of Electromagnetic Theory. Addison-Wesley, Reading (1979)
14. Sciama, D.W.: MNRAS **113**, 34 (1953)
15. Weinberg, S.: Gravitation and Cosmology. Wiley, New York (1972)